Stabilizing unstable orbits by slow modulation of a control parameter in a dissipative dynamic system

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Stabilization of unstable periodic orbits embedded in chaotic or nonchaotic attractors is achieved by slow nonfeedback periodic modulation of a control parameter, without crossing bifurcation boundaries. Also, it is shown that a step change in the parameter inside one periodic regime can destabilize the system, bringing it to an unstable limit cycle dependent on the amplitude and phase of the parameter change. Periodic modulation of the control parameter with a period smaller than the average time spent on the unstable orbit stabilizes this orbit. The effect is demonstrated in a loss-driven CO_2 laser with periodically modulated cavity detuning. The results of numerical simulations are in a good agreement with experimental results. [S1063-651X(98)03404-7]

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I. INTRODUCTION

Unstable orbits are very important characteristics of dynamic systems. They provide significant information about the basins of attraction of different attractors in phase space and help to study their topological invariants [1,2]. Many methods of controlling chaotic and nonchaotic dynamics are based on stabilization of unstable periodic orbits. Indeed, the original chaos control algorithm developed by Ott, Grebogi, and Yorke [1] consists in stabilization of a chosen periodic orbit embedded within a chaotic attractor by applying small time-dependent perturbations to one of the adjustable system parameters. The method is based on the fact that a chaotic attractor can be represented as an infinite number of unstable periodic orbits [3]. This method and its variations have been successfully applied to mechanical [4], electronic [5], fluid [6], chemical [7], laser [8], and biological systems [9]. The principles of the Otto-Grebogi-Yorke (OGY) method have been used also to stabilize unstable periodic orbits in nonchaotic systems. Recently, Christini and Collins [10] proposed the employment of external noise for destabilizing stable periodic motion and in such a way sending the system state to a desired unstable orbit where the OGY control technique can be applied.

Another approach in controlling nonlinear dynamics, which, as distinct from the OGY control method, does not require either a permanent analysis of the system behavior or a feedback loop, is the application of a weak periodic perturbation on some system parameter at a *resonant* frequency [11] related to a driving force [12]. However, it has been shown recently [13] that the subharmonic parametric perturbation produces a new dynamical system by splitting the original attractor into two new ones. As a rule, the system is attracted to a new attractor with lower complexity, but not to an unstable orbit in the original attractor.

Although the notion of a chaotic attractor is relatively recent, the settling of transients in dissipative systems is a common and familiar behavior. The decaying transients occur in an approach to a stable fixed point or limit cycle. It has been shown recently [15] that during transients occurring after a short impulsive perturbation of a system parameter, the system can evolve towards an unstable orbit. More recently, a Q-switching technique has been proposed for targeting unstable orbits in a class-B laser [14]. Moreover, both the Q-switching and short impulsive excitation, depending on their phase, can bring the system to other stable coexisting attractors [15,14]. Chaotic transients in a loss-driven CO_2 laser after a sudden switch of control parameters between values corresponding to two different dynamic regimes were experimentally studied by Papoff *et al.* [16]. Later, transient statistics after switching a small resonant parametric perturbation was investigated by Meucci *et al.* [17].

In the present paper we study a method of stabilizing unstable periodic orbits in a dissipative system by using a *slow* periodic modulation of a control parameter without crossing bifurcation boundaries (i.e., when the control parameter is modulated between values corresponding to one dynamic regime, e.g., inside a period-2 orbit or chaos). The term ''slow'' means that the frequency of the control modulation is much smaller than the characteristic frequency of the system, e.g., the driving frequency. We study the relation of this effect with relaxation processes occurring after a step parameter change. The method is verified in a loss-driven CO_2 laser with the use of cavity detuning as a control parameter that is periodically modulated. We investigate how the stabilizing effect depends on the amplitude and frequency of the control modulation as well as on the sensitivity of the system to the parameter change.

The main advantage of our technique over the method of tracking unstable steady states by large-amplitude periodic parametric modulation proposed recently by Vilaseca and co-workers [18,19] is that in our approach the proper choice of modulation frequency and amplitude allows the stabilization of unstable orbits without large shifts of the parameter. In this sense, it is not necessary for the system to pass forward and backward through various bifurcations, from regimes where the orbit to be tracked is stable into its unstable parameter range. This makes our method applicable to systems that cannot be shifted very far from their natural parameter range to access a bifurcation point.

The plan of the paper is as follows. In Sec. II we describe very briefly a model for the loss-driven CO_2 laser and provide several numerical solutions in order to illustrate the effectiveness of the method. Some are to demonstrate the difference in transients occurring when cavity detuning rapidly switches from lower to higher values and back in a period-2 regime. Special attention is focused on the influence of the switching phase on the duration of transients. The other examples show how the sensitivity of the system to the change of the control parameter has an effect on the stabilization of unstable orbits. Dynamical hysteresis, which takes place when detuning periodically increases and decreases, is studied in the laser response. We show how this hysteresis depends on the modulation frequency of detuning. We also investigate how the modulation amplitude of the control parameter influences the stabilization of unstable orbits. At the end of the section we demonstrate the applicability of the method to the stabilization of periodic orbits embedded within a chaotic attractor.

In Sec. III we describe our experimental results, which confirm the results of the numerical simulations. Finally, in Sec. IV we present the main conclusions.

II. NUMERICAL SIMULATIONS

A. Model

The operation of a loss-driven CO_2 laser can be described by a system of equations based on the well-known four-level model [20]. In addition to five differential equations for the laser intensity, the populations of the upper and lower lasing levels, and the global populations of the two manifolds of rotational levels, we introduce three equations that take into account the energy exchange between CO_2 and N_2 molecules, and the rates of vibrational-vibrational energy exchange within symmetric and asymmetric modes of the CO_2 molecule.

Let the active medium of the CO_2 laser before switching on the electric discharge to be a mixture of the gases CO_2, N_2, and He. For the sake of definiteness, we shall consider a single-mode lasing within a vibrational-rotational transition of the 00^01-10^00 channel. The CO_2 laser with modulated losses can be described by the system of equations [21]

$$\frac{dN_1}{dt}=\beta_1 n_e N_0 - W_{10}N_1 + W_{21}N_2 + B(\nu)u(n_2^j - n_1^j), \quad (1)$$

$$\frac{dN_2}{dt}=\beta_2 n_e N_0 + W_{NC}N_N(N_0M_1 - N_2M_0) - W_{21}N_2 - B(\nu)u(n_2^j - n_1^j), \quad (2)$$

$$\frac{dM_1}{dt}=\beta_3 n_e M_0 + W_{NC}N_C(N_2M_0 - N_0M_1), \quad (3)$$

$$\frac{dn_1}{dt}=B(\nu)u(n_2^j - n_1^j) - V_1(n_1 - N_1), \quad (4)$$

$$\frac{dn_2}{dt}=B(\nu)u(n_2^j - n_1^j) - V_2(n_2 - N_2), \quad (5)$$

$$\frac{dn_1^j}{dt}=B(\nu)u(n_2^j - n_1^j) - V_R(n_1^j - F_1^j n_1) - V_1(n_1^j - F_1^j N_1), \quad (6)$$

$$\frac{dn_2^j}{dt}=B(\nu)u(n_2^j - n_1^j) - V_R(n_2^j - F_2^j n_2) - V_2(n_2^j - F_2^j N_2), \quad (7)$$

$$\frac{du}{dt}=c\mu[\kappa(\nu)y - k]u. \quad (8)$$

Here N_0, N_1, and N_2 are the relative quasiequilibrium populations of the vibrational 00^00, 10^00, and 00^01 levels of CO_2; M_0 and M_1 are the relative populations of the fundamental and first exited vibrational level of N_2; n_1 and n_2 are the relative quasinonequilibrium populations of the vibrational 10^00 and 00^01 levels of CO_2 [22]; n_1^j and n_2^j are the relative populations of lower and upper laser rotational sublevels [23]; n_e is the free-electron density in the active medium; W_{21} and W_{10} are the effective rates of collisional relaxation in 00^01-10^00 and 10^0 0-00^00 channels; V_R is the rotational relaxation rate; V_1 and V_2 are the vibrational relaxation rates that describe the relaxation of ''instantaneous'' populations n_1 and n_2 to their quasiequilibrium values N_1 and N_2; W_{NC} is the exchange rate of the vibrational excitation from N_2 to CO_2; β_1, β_2, and β_3 are the pumping rates of N_2 and the lower and upper levels of CO_2 in the electric discharge; N_C and N_N are the volume density of CO_2 and N_2; F_1^j and F_2^j are the normalized Boltzmann functions determining the part of molecules in the corresponding rotational sublevels in thermodynamic equilibrium: $n_1^j = F_1^j n_1$, $n_2^j = F_2^j n_2$; $B(\nu)$ and $\kappa(\nu)$ are the Einstein coefficient and specific gain coefficient at the lasing frequency ν; l_m and l_a are the lengths of the loss modulator and active medium; c is the speed of light in the active medium; μ is the packing coefficient for the active medium in the cavity; u is the average radiation density; $y = n_2^j - n_1^j$ is the population inversion.

The loss coefficient of the laser cavity is described by the expression

$$k = k_0 + \Delta k[1 - \cos(2\pi f_0 t)], \quad (9)$$

where k_0 is the loss coefficient without modulation, Δk and $f_0 = 1/T$ are the (driving) amplitude and frequency of loss modulation, and T is the period of loss modulation. The simulations are performed for parameters appropriate for the following experimental situation. The active medium is a mixture of $CO_2:N_2:He = 1:1:8$ at a pressure of 15 torr. The laser operates on a single mode at the $10P20$ line. The cavity length is 2 m, the length of the active medium is 1.8 m, and $f_0 = 110$ kHz. Other parameters are varied in numerical simulations.

Several previous theoretical and experimental works [19,24] have shown very high sensitivity of the dynamics of a class-B laser to changes in cavity detuning that leads to the corresponding changes in the lasing frequency ν. Therefore, we choose just this parameter as a control parameter in our numerical and experimental research. We define the cavity detuning as $\delta = (\nu - \nu_0)/\gamma$, where ν_0 is the central frequency

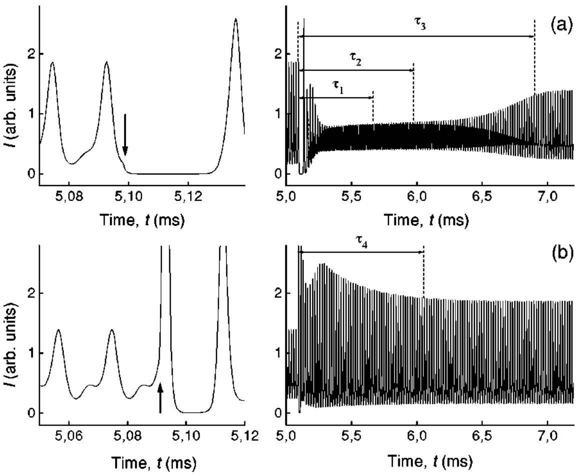

FIG. 1. Numerical transients at the (a) forward and (b) backward step switching of the cavity detuning between $\delta_1 = 0.525$ and $\delta_2 = 0.535$. $k_0 = 6\times10^{-3}$ cm^{-1} and $\Delta k = 7\times10^{-6}$ cm^{-1}. The arrows indicate the moments of the detuning switch. Note the expanded horizontal scale on the figures of the left-hand column. Transient times τ_1, τ_2, τ_3, and τ_4 are defined in the text.

and γ is the half-width at half maximum of the Lorentzian gain line shape. The period-2 bubble appears with detuning between $\delta_{01} = 0.510$ and $\delta_{02} = 0.542$ at $k_0 = 6\times10^{-3}$ cm^{-1} and $\Delta k = 7\times10^{-6}$ cm^{-1}.

B. Transients after step change of the control parameter

Detailed investigations of transients occurring after step change in detuning inside a period-2 domain between δ_{01} and δ_{02} show that the duration of transients after forward and backward switching of detuning (from $\delta_1 = 0.525$ to $\delta_2 = 0.535$ and back) are different. The time evolution of the laser intensity I, which is proportional to the instantaneous value of the radiation density $u(t)$, is shown for illustration in Fig. 1. From these examples of transients after forward and backward switching of detuning one can see that after step increasing detuning from δ_1 to δ_2 [Fig. 1(a)], the lasing switches off for a short time and then the system evolves towards an unstable period-1 orbit before it reaches another stable period-2 orbit. To characterize transients we distinguish in Fig. 1(a) three time intervals from the moment of the parameter change: the time of running into the period-1 orbit (τ_1), when the intensity difference between two successive peaks is minimum, i.e., when the system approaches closest the period-1 orbit; the time of leaving the period-1 orbit (τ_2) when the peak intensity exceeds by 10% that at the previous peak; and the time of reaching a new period-2 regime (τ_3) when the maximal intensity reaches 0.9 of that at the new stable period-2 regime. When detuning decreases, i.e., switches from δ_2 to δ_1 [Fig. 1(b)], the lasing never switches off immediately and the period-1 regime appears in the laser response only within a very narrow phase range. In this case, the system relaxes with time τ_4 when the maximum intensity reaches 1.1 of that at the new stable period-2 regime. Thus τ_3 and τ_4 characterize the total durations of transients after forward and backward switching.

Comparing Figs. 1(a) and 1(b), one can see that the duration of transients is shorter when detuning decreases, i.e., when the parameter changes so that the maximum intensity at the new stable state is greater. As distinct from a Q-switched laser [14], where the laser is switched off, so that it ''forgets'' its previous history, and is switched on again, the step change of the parameter leads to the interruption of lasing for a relatively short time (2–3 T) [see the left-hand side of Fig. 1(a)] and the laser still ''remembers'' the phase of the parameter switch. Physically, when the lasing is absent, the inversion is progressively increasing and finally a giant pulse is lased and damped oscillations destabilize the system. This destabilization brings the system state closer to a desired unstable orbit. The maximal intensity in the first giant pulse depends on the duration of the no-lasing state, which is determined by the switching phase and the amplitude of the parameter change. The step change in the parameter can be considered as a sudden kick of the system that is similar to the influence of an external δ-function impact. As was shown in [15], the kick moves the system state to another point of its phase space. In addition to the simple movement of the current system state, the step change of the parameter results in a slight change of the structure of the phase space itself.

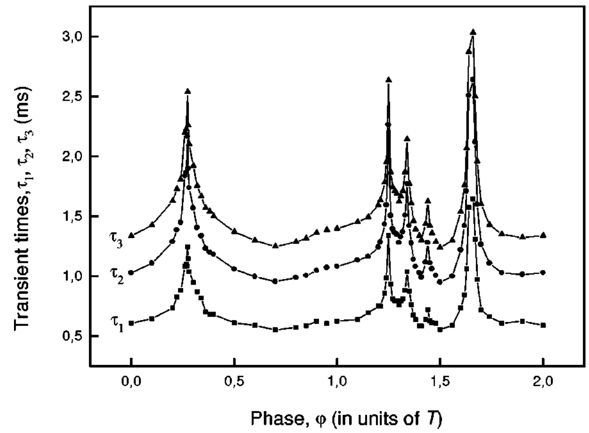

FIG. 2. Phase dependence of transient times after the step switch of the cavity detuning from $\delta_1 = 0.525$ to $\delta_2 = 0.535$.

Figure 2 shows the phase dependence of the duration of transients after the switch of detuning from δ_1 to δ_2 within the period-2 domain situated between δ_{01} and δ_{02}. As can be seen from the figure, all times correlate with each other. The period-1 orbit is targeted at any phase of switching, but the duration of transients displays a strong dependence on the switching phase φ. One can see from Fig. 2 that there are five maxima at which targeting of the unstable period-1 orbit is optimal. The first maximum (at $\varphi = 0.27$ T) corresponds to the case when the detuning switches in the range of minimal laser intensity, i.e., between lasing spikes. At this moment, the gain coefficient is close to the loss coefficient and the laser is very sensitive to the parameter change. The switch of δ at the moments corresponding to the beginning of the nonlinear stage in lasing pulse leads to the appearance of three close situated maxima in Fig. 2 (at $\varphi = 1.26$, 1.35, and 1.41 T). Finally, the last maximum (at $\varphi = 1.7$ T) corresponds to switching δ at the moment when the laser intensity is maximal. Additional calculations show that the number of targeting maxima and their positions depend on the shape of lasing pulses. Absolute maximal transient times $\tau_3^{max} \approx 3.0$ ms,

$\tau_2^{max} \approx 2.6$ ms, and $\tau_1^{max} \approx 1.6$ ms are achieved at the switching phase $\varphi = 1.7$ T for the conditions of Fig. 2.

Additional investigations show that after parameter switch, during transients, the system can also evolve towards another stable coexisting attractor at certain switching phases. Similar behavior has been observed under the influence of targeting δ pulses [15] and Q switching [14]. The targeting of coexisting attractors depends also on the amplitude of the detuning change, i.e., on $\delta_{21} = \delta_2 - \delta_1$.

C. Stabilizing an unstable period-1 orbit within a period-2 domain

In this subsection we shall show how periodic modulation of the system parameter with a period smaller than the average (over all switching phases) duration of the unstable period-1 orbit in transients after step changing the control parameter (τ_2 in Fig. 2) leads to stabilization of the period-1 orbit. The cavity detuning can be expressed as

$$\delta = \delta_0 + \Delta\delta[1 - \cos(2\pi f_1 t)], \qquad (10)$$

where δ_0 is the initial detuning from the center of the gain contour without the control modulation and $\Delta\delta$ and f_1 are the amplitude and frequency of the control modulation. We consider a slow modulation of the control parameter (i.e., $f_1 \ll f_0$) and therefore we neglect a phase difference between the control and driving signals.

Figure 3 demonstrates the stabilizing effect of periodic modulation of detuning on the laser initially operating in a period-2 regime. One can see in Figs. 3(a) and 3(b) two hysteresis loops corresponding to stroboscopic measuring maxima of the laser intensity I at time intervals T, with increasing (forward) and decreasing (backward) detuning. From these figures it is seen that with increasing control frequency, these loops diverge (converge) in the range of small (large) detuning. With a further increase f_1, they overlap [Fig. 3(c)] and finally merge into a single loop [Fig. 3(d)]. This single loop corresponds to the period-1 regime. Thus there is a minimal control frequency f_{st} at which the unstable orbit is completely stabilized. This frequency depends, of course, on the parameters of the control modulation, which will be discussed in detail in Sec. II D.

The stabilizing effect can be quantitatively estimated in terms of the difference between the intensities for the upper and lower loops at the forward (ΔI_+) and backward (ΔI_-) sweep of detuning at fixed δ. The behavior of these differences versus the control frequency at $\delta = 0.53$, shown in Fig. 4(a), is complex. One can see that ΔI_+ has a maximum at $f_1 \approx 400$ Hz, where the difference between ΔI_+ and ΔI_- is maximal; at $f_1 \approx 500$ Hz they intersect; with a further increase of f_1 they diverge; then at $f_1 \gtrsim 800$ Hz they converge; finally, at $f_1 \gtrsim 1.1$ kHz the curves merge, i.e., this means that the period-1 orbit is stabilized. Similar behavior is observed at other fixed δ.

For a quantitative characterization of the dynamical hysteresis, we introduce the value of $H = \Delta I_+ - \Delta I_-$, which yields the difference between the maximal intensities at fixed δ at forward and backward sweeping. The value of H versus the control frequency at $\delta = 0.53$ is shown in Fig. 4(b). This dependence has two extrema. The maximum corresponds to

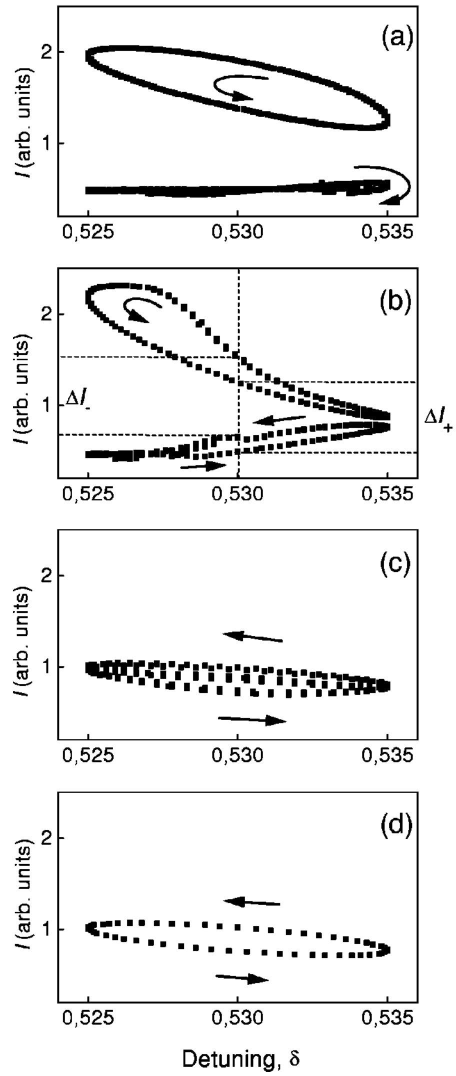

FIG. 3. Numerical stroboscopic diagrams showing dynamical hysteresis loops at different control frequencies (a) $f_1 = 200$ Hz, (b) $f_1 = 500$ Hz, (c) $f_1 = 1$ kHz, and (d) $f_1 = 2$ kHz. $\delta_0 = 0.525$ and $\Delta\delta = 0.005$. The arrows indicate the direction in the change of detuning. (b) The definitions of ΔI_+ and ΔI_- are shown.

the maximal divergences of the hysteresis loops shown in Fig. 3, the loops begin to merge at the range of negative H ($f_1 \gtrsim 500$ Hz), and the period-1 orbit is stabilized at $H = 0$ ($f_1 \gtrsim 1.1$ kHz), when only a single loop remains.

Additional calculations show that comparing the duration of transients after a step change in detuning with the period of the control modulation, we can make the following conclusions. (i) Increasing the control frequency until the period of the modulation becomes equal to the maximal duration of transients, i.e., until $1/f_1 = T_1 = \tau_3^{max}$, the maximal intensity increases and hysteresis loops diverge. (ii) Suppression of the period-2 regime is realized when $T_1 \lesssim \tau_3^{max}$. (iii) The period-1 regime appears when $T_1 \lesssim \tau_2^{max}$, i.e., when the period of the control modulation is smaller than the maximal time of the system's stay on the period-1 orbit. (iv) Stabilization of the period-1 orbit takes place when T_1 is shorter than the average time of the system's stay on the period-1 orbit over all phase range, i.e., average τ_2.

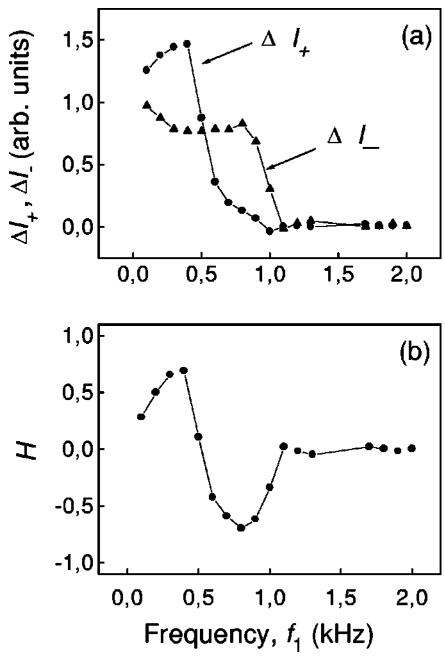

FIG. 4. (a) Difference between hysteresis loops and (b) hysteresis parameter H versus modulation frequency at fixed detuning $\delta = 0.53$ (for definition see the text). The conditions correspond to that of Fig. 3. Two extrema in (b) display convergence and divergence of hysteresis loops with the control frequency.

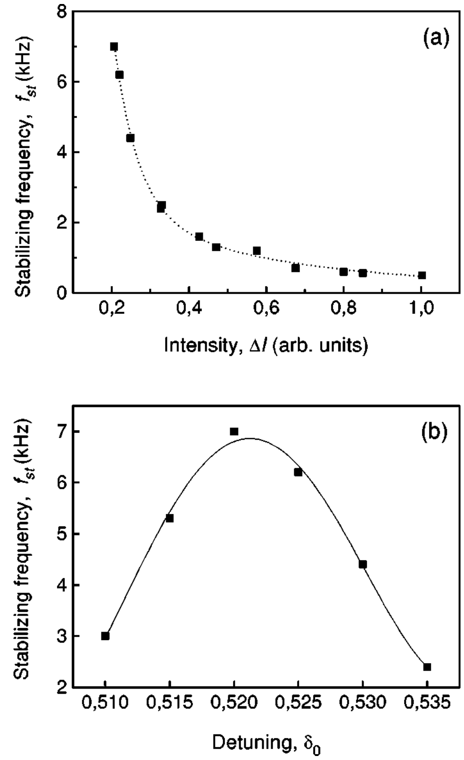

FIG. 5. Minimal stabilizing frequency versus (a) the asymptotic change in laser intensity and (b) the initial value of detuning. Detuning is modulated inside the period-2 domain. In (b) $\Delta\delta = 0.005$.

D. Sensitivity to the change of control parameter

The stabilization effect depends on the amplitude $\Delta\delta$ and frequency f_1 of control modulation as well as on the initial detuning δ_0 [see Eq. (10)]. Generally, the efficiency of stabilization is determined by the sensitivity of the system to the parameter change. In the absence of detuning modulation, the laser with cavity loss modulation displays the period-2 cycles both for $\delta_1 = \delta_0$ and for $\delta_2 = \delta_0 + 2\Delta\delta$, with maximal peak intensities I_1 and I_2, respectively. The sensitivity can be quantitatively characterized by the dependence of f_{st} on $\Delta\delta$ or on $\Delta I = I_1 - I_2$. However, ΔI is not a linear function of $\Delta\delta$ and for generalization of the results we study the dependence on ΔI because this value characterizes the difference in the system state either at two fixed values of the control parameter or when the parameter changes adiabatically between these two boundary values. Moreover, the stabilization can be achieved by modulating a control parameter other than δ in a different nonlinear system.

The minimal stabilization frequency as a function of ΔI is shown in Fig. 5(a). The detuning is modulated inside the period-2 domain situated between δ_{01} and δ_{02}. The values of ΔI are determined by using different $\Delta\delta$. The data in Fig. 5(a) are well fitted by the double exponential curve

$$f_{st} = \alpha_1 \exp[(\Delta I_0 - \Delta I)/\beta_1] + \alpha_2 \exp[(\Delta I_0 - \Delta I)/\beta_2], \tag{11}$$

where the constants $\Delta I_0 = 0.196$, $\alpha_1 = 6$, $\alpha_2 = 2$, $\beta_1 = 0.065$, and $\beta_2 = 0.676$. Thus the stabilizing frequency decays exponentially with ΔI. Extrapolation of this dependence to $\Delta I \to \infty$ leads to $f_{st} \to 0$. This means that at very high changes in the laser response, the stabilization is possible at very slow modulation of the parameter, while at very small ΔI, the stabilizing frequency should be very high. The upper range in f_{st} is bounded by the commensurability of the control frequency with the driving one. An interaction between these two frequencies can result in phase locking that leads to additional hysteresis effects.

The sensitivity of the system to the parameter change depends on the distance from the bifurcation point or, in other words, on the vicinity of the system to the desired unstable orbit. Figure 5(b) displays the evolution of the stabilizing frequency versus the distance from period-doubling bifurcation points (remember that the period-doubling bifurcations are observed at $\delta_{01} = 0.510$ and $\delta_{02} = 0.542$). One can see that closer to the bifurcation point, the laser is more sensitive to the change in the control parameter, i.e., the period-1 orbit is stabilized at a lower control frequency.

Additional simulations show that the sensitivity is different when detuning is varied within a period-2 domain situated at the center of the gain contour. Obviously, closer to the maximum gain, the laser is less sensitive to detuning and the control frequency needed to stabilize an unstable orbit is higher, i.e., for targeting an unstable orbit the detuning should change faster.

E. Stabilizing periodic orbits embedded within a chaotic attractor

Stabilization of periodic orbits can be achieved also when detuning is modulated within a chaotic domain that appears at certain detuning when the driving amplitude increases up

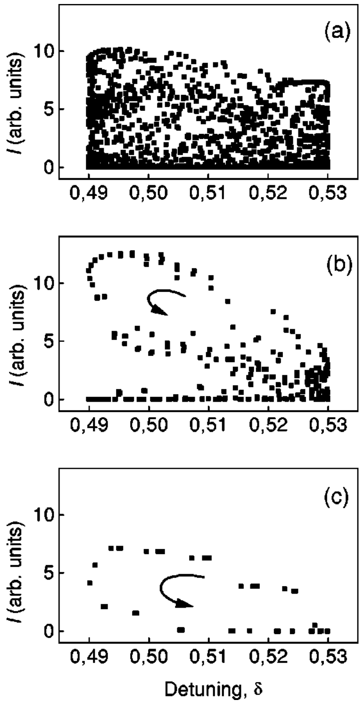

FIG. 6. Numerical bifurcation diagrams showing stabilization of periodic orbits within a chaotic attractor with increasing control frequency (a) $f_1=200$ Hz, (b) $f_1=1$ kHz, and (c) $f_1=6$ kHz. $k_0=6\times10^{-3}$ cm^{-1}, $\Delta k=2.4\times10^{-5}$ cm^{-1}, $\delta_0=0.49$, and $\Delta\delta=0.02$. The arrows indicate the direction in the change of detuning.

to $\Delta k=2.4\times10^{-5}$ cm^{-1}. Figure 6 demonstrates inhibition of chaos when the control modulation is applied. One can distinguish in Fig. 6(b) periodic regimes that appear with increasing f_1. At $f_1=1$ kHz, a period-2 regime is reached at low detuning [left-hand side of Fig. 6(b)], while at $f_1=6$ kHz only a single period-1 orbit remains over the whole range of detuning [Fig. 6(c)]. Dynamical hysteresis is clearly seen in Figs. 6(b) and 6(c).

III. EXPERIMENT

The experiments were carried out with a single-mode CO_2 laser with modulated losses via an acousto-optic modulator. The experimental setup is similar to that described in previous work [19]. The *driving* electric signal $V_d=A_d\sin(2\pi f_d t)$ at frequency $f_d=105$ kHz and amplitude A_0 is applied to the modulator providing the time-dependent cavity losses in accordance with Eq. (9). The *control* electric signal $V_c=A_c^0+A_c(t)\equiv A_c^0+\Delta A_c[1-\cos(2\pi f_c t)]$, with a constant bias voltage A_c^0 and modulation amplitude ΔA_c and frequency f_c, is used to tune the output mirror with a piezotranslator. This signal produces appropriate changes in cavity detuning in accordance with Eq. (10). The frequency of relaxation oscillations at the center of the gain line is about 108 kHz. This value is estimated from averaged power spectra when noise is applied to the modulator. Other parameters are the same as those used in simulations.

Figure 7 shows an experimental bifurcation diagram with cavity detuning as a control parameter. This diagram is obtained by applying a ramp signal to the piezotranslator with a

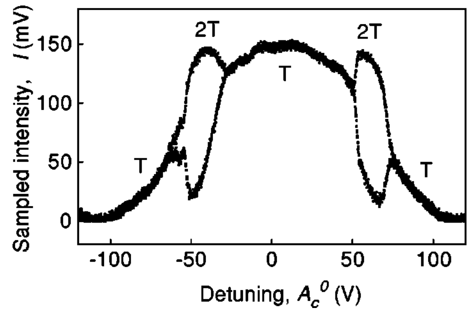

FIG. 7. Experimental birfurcation diagram with detuning A_c^0 as a control parameter. $A_d=7$ V and $A_c(t)=0$. Here the detuning is ramped at a constant rate for 40 ms.

duration of 40 ms. One can see that within certain ranges of detuning the laser operates in a period-2 regime.

Examples of experimental transients after a step change of detuning are shown for illustration in Fig. 8. A small change of detuning inside the period-2 domain (right-hand side of Fig. 7) does not lead to destabilization of the system and a period-1 orbit is not targeted [Fig. 8(a)], while the enhancement of the detuning range allows one to target a period-1 orbit in transients, as can be seen in Fig. 8(b). In this case, the transients are larger (5.6 ms) than in the previous case. One can compare in Fig. 8(a) the duration of transients at forward and backward switching of detuning, which are equal to 3.8 and 2 ms, respectively. Thus the transient time is larger when detuning changes from a lower to a higher value that agrees with simulations.

Following Eq. (10), we apply a sinusoidal modulation to cavity detuning so that the system remains inside the period-2 domain. Experimental bifurcation diagrams shown in Fig. 9 demonstrate the effect of the control modulation at different frequencies. One can see that with increasing f_c, the period-2 regime is progressively suppressed [Figs. 9(b) and 9(c)], the period-1 orbit appears in the laser response [Fig. 9(c)], and complete stabilization is achieved at $f_c=2$ kHz [Fig. 9(d)]. Comparing these diagrams with numerical ones shown in Fig. 4, one can see a good qualitative agreement.

Periodic modulation of detuning inside a chaotic domain,

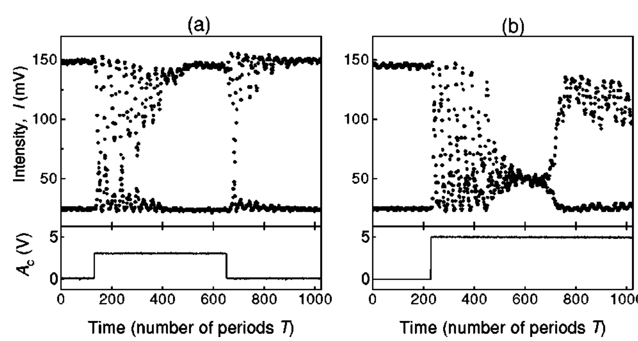

FIG. 8. Experimental stroboscopic transients after different step changes in cavity detuning. The voltage A_c applied to the piezotranslator that tunes the cavity mirror is shown in the lower part of the figures. (a) Difference in transients when detuning switches on and off. (b) Transients led to an unstable period-1 regime at higher A_c. $A_d=7$ V and $f_c=0$.

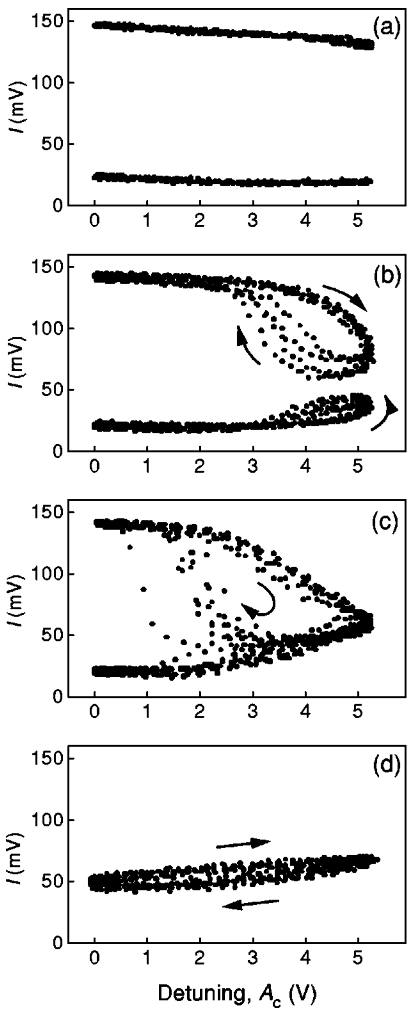

FIG. 9. Experimental stroboscopic diagrams showing stabilization of the period-1 orbit within the period-2 attractor at different control frequencies (a) $f_c = 200$ Hz, (b) $f_c = 500$ Hz, (c) $f_c = 1$ kHz, and (d) $f_c = 2$ kHz. $A_d = 7$ V.

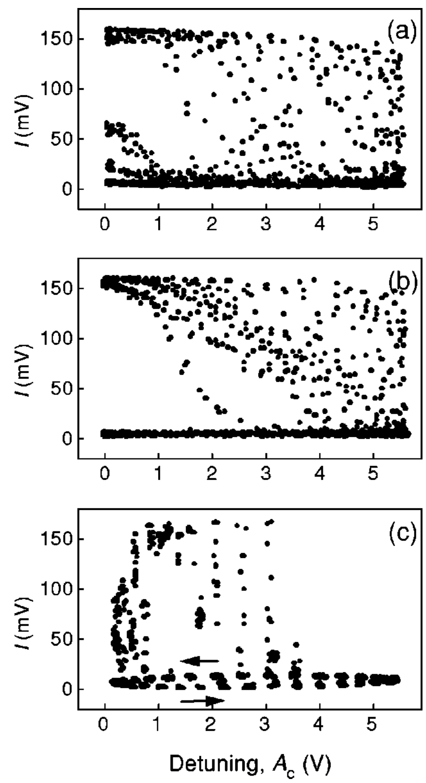

FIG. 10. Experimental stroboscopic diagrams showing the appearance of a periodic structure in the chaotic attractor at different control frequencies (a) $f_c = 200$ Hz, (b) $f_c = 500$ Hz, and (c) $f_c = 3$ kHz. $A_d = 10$ V.

which appears at a higher driving amplitude ($A_d = 10$ V), results in an alternation of chaotic and periodic regimes (Fig. 10). With increasing f_c, the range of the existence of the periodic regimes increases [compare Figs. 10(a) and 10(b)] and finally, the period-1 orbit is stabilized at a certain range of detuning [Fig. 10(c)]. Although we did not manage to stabilize the period-1 orbit over the whole detuning range because of the laser instabilities, these results confirm our numerical simulations and demonstrate the validity of the present approach for stabilizing periodic orbits embedded within a chaotic attractor.

IV. CONCLUSIONS

In this work we have numerically shown that a slow periodic modulation of a control parameter can stabilize unstable periodic orbits embedded within chaotic or nonchaotic attractors. We have studied how the duration of transients after a step change of the control parameter depends on the phase of the parameter switch. We have demonstrated that the change of the parameter can direct the system's trajectory towards the attracting stable manifold of the desired unstable periodic orbit. The best conditions for targeting unstable orbits have been investigated in detail. The introduction of periodic modulation of the control parameter with a period shorter than the time of the existence of the unstable orbit during transients does not allow the system to leave the unstable orbit and hence this leads to its stabilization. Once this orbit is targeted, the system remains on it until the control is switched off.

We have applied this method to a loss-driven CO_2 laser with periodically modulated cavity detuning. We have studied the dynamical hysteresis appearing in the laser response when detuning is periodically modulated. The hysteresis loops display divergent or convergent behavior depending on the control frequency. We have investigated how the efficiency of stabilization depends on the system's sensitivity to the parameter change. Minimal stabilization frequency decays exponentially with the increasing difference between adiabatic laser intensities at the boundary values of detuning. The technique described in this work requires no knowledge of the underlying system behavior. The results of numerical simulations are in good agreement with experiments.

Although the method proposed was applied to a laser, we think that a similar approach can be implemented to different dissipative dynamic systems whose behavior depends strongly on parameters. For instance, some biological and medical experiments can be considered from this point of view; in particular, dynamical diseases tend to exhibit transient behaviors and the severity of their symptoms varies over time [25]. As an example, medical experiments on fluctuations in tremor and respiration in patients with Parkinson's disease [26] may be explained in the frame of present work. Parkinson's disease is a chronic neurodegenerative disorder characterized by an involuntary (chaotic) movement of a part of the body. The correlation between tremor amplitude and respiration rate has been observed: the tremor amplitude decreases with respiration rate after transients of several seconds. These changes may be a consequence of stabilizing periodic orbits due to the alternation in one (or more) significant physiological variable(s).

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